Acta Technica 62, No. 3B/2017, 419–428 

# A modification composite scale of cloud model using in consistency of its judgment matrix for decision making[1]

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**Abstract.** In real situation, uncertain circumstances will possibly lead to various practical problems, especially when methods like the analytic hierarchy process (AHP) are combined with subjective evaluations. This paper proposes a composite scale model with consistency of its judgment matrix, with the improved cloud model, index scale and $-n \sim n$ scale being integrated and cloud index composite scale as the theoretical foundation. In order to resolve the Multi-attribution Decision-making Problem (MADPMP) of linguistic assessment information, this paper also presents an improved scheme, which can achieve the ideal compatibility of the objective judgment matrix and subjective evaluations, and which considers limitations of the highest importance level of index scale method and correspondence of language description & number. In addition, examples are timely demonstrated in this paper, so as to support the new algorithm. Results have verified the practicality of the index scale and superiority of the improved scheme. In this way, the feasibility of the algorithm is also confirmed.

**Key words.** Multi-attribution decision-making, analytic hierarchy process (AHP), index scale, cloud model.

## 1. Introduction

Analytic hierarchy process (AHP), a very practical method for multi-criteria decision making, is now most frequently used in many areas [1], [2]. In this method, objective mathematics is employed to deal with the subjective and different tastes of a collective or an individual, so as to make reasonable decisions. In the hierarchical analysis provided by Saaty, a person (an expert, for example) will be required to offer his/her ratios $a_{ij}$ for each pairwise comparison between issues (alternatives,

[1]This work is supported by Education Department of Fujian Province under project number JA15556, Youth Project of Ningde Normal University (2015Q08).

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candidates, etc.) $A_1$, $A_2$ ... $A_\mathrm{n}$ for each criterion (objective) in a hierarchy and also between the criteria. To stress differences, a range of numbers will be adopted to compare the importance and dominance of an element with that of others in terms of criterion or property [3], [4]. Saaty's way to stress difference is to adopt fundamental nine-scale measurement, in which there are numbers for the ratios from set $\{1, 2 \ldots 9\}$. Its level of judgments range from equal, moderately more, strongly more, very strongly more to extremely more [5], [6]. However, the language description does not correspond to the numerical values of the scale division in many aspects.

The first volatility is the moderate discrepancy of numerical values and linguistic description [7]. Compare $a_1$ and $a_2$ from the perspective of the sizes (usually importance); $a_{12} = 3$ shows that $a_1$ is twice larger (important) than $a_2$. Nevertheless, relevant linguistic description suggests that $a_1$ is only a little larger (slightly more important) than $a_2$. The second volatility is that for different people (experts, judges, for example), even the same linguistic description will mean different [9]. "Moderately more" may mean 1.2 or something like that for some people. However, there is a possibility that it will mean 2 or more in others. Too often, affected by various environments, people will attach different meanings to even the same linguistic description. It is fairly difficult to avoid volatility in AHP and in group decision-making, for there are different linguistic descriptions, especially the inconsistency between the language description and the actual nine-scale division measurement. In real situation, it is far from impossible to achieve unified linguistic term sets for different people and different issues. Therefore, a uniform scale measurement needs to be applied to find the differences. However, assigning pairwise comparisons usually cannot be achieved without uncertainties, considering human subjective evaluations.

Li Deyi's proposed cloud model is a qualitative and quantitative uncertainty model based on stochastic mathematics and fuzzy mathematics [8]. Universality of normal distribution and Gaussian membership function constitutes the theoretical basis of the normal cloud model universality [9], [10]. Therefore, through the analysis of the distribution characteristics of cloud model and utilization of advantages of expectations and membership functions, in this paper, we propose a new composite scale (referred to as the cloud index composite scale) which combines the index scale function and the scale advantage, and establishes an evaluation model based on the new compound scale. Meanwhile, considering the poor consistency problem of judgment matrix caused by subjective assessment, and according to the relationship between sensation judgment matrix and reciprocal judgment matrix, this paper proposes the completely consistent construction sensation judgment matrix and the calculation of index weight algorithm with it.

The rest of this paper is organized as follows: The second section introduces theories of scale and the concept of cloud model. The new composite scale, the description method of the cloud index scale and the basis concerning the choice of the importance ratio parameters $a$ are presented in section three. Then, the consistency check of sensation judgment matrix is adopted to propose a correct sensation judgment matrix algorithm towards the problem of poor consistency in objective judgment matrix in next section. Finally, section 5 concludes the paper.

## 2. Building a new of composite scale

Index scale provides the basis for the value change of compound scale, but features in numerical one-way growth [5], [9], [10]. However, the state of attribute decision attributes things affects psychological stimulation of the decision makers in both good and bad ways, if index scale is only used to evaluate attributes, then semantic deviation will occur, as shown in Fig. 1.

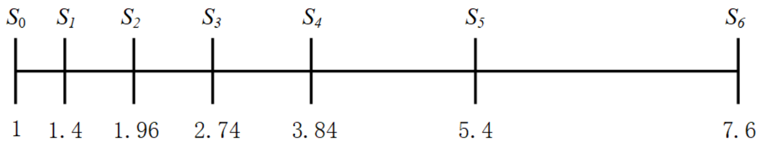


Fig. 1. The application of pure index scale to evaluation scale. Remarks: $n = 7$; $a = 1.4$; $S_0$ very poor; $S_1$ poor; $S_2$ fair poor; $S_3$ common; $S_4$ fair good; $S_5$ good; $S_6$ fair good

As is shown in Fig. 1, minimum distance has occurred between 'very poor' and 'poor', which do not conform to the actual decision-making case. What is worse, there are deep inconsistencies between 'good' and 'fair good'. Therefore, we need to absorb the merits of $-n \sim n$ scale, achieve two-way level geometric growth of the scale point value, and fit real psychological physical values in the evaluation process, so as to meet the demand for scale and principle of geometric assignment and phase by leaps. Composite evaluation scale expression is given below [8], [10]:

$$a^{y_i} = \begin{cases} a^{\left(\delta_i^{\max} - \exp\left(\frac{(x_i - Ex_i)^2}{2(En)}\right)\right)} & x_i > Ex_i \geq 0 \\ a^{\left(\frac{\delta_i^{\max} + \beta_i^{\min}}{2}\right)} & x_i = Ex_i \\ a^{\left(\beta_i^{\min} + \exp\left(\frac{(x_i - Ex_i)^2}{2(En)}\right)\right)} & 0 < x_i < Ex_i \end{cases} . \tag{1}$$

where $a^{y_i}$, $i = 1, 2, \ldots, g$ is defined as cloud index composite scale. In addition, there is a symmetry between value $a^{y_i}$, when the scale is $-n \sim 0$ and $0 \sim n$, which will not be repeated here.

Next, further elaboration of value $y_i$ in cloud index composite scale $a^{y_i}$ is given. Assume that there are $i$ remark sets in qualitative evaluation, and each remark has a bilateral constraint interval $\left[\beta_i^{\min}, \delta_i^{\max}\right]$, based on the $3En$ principle of cloud model, remark set cloud model can be determined by [10]

$$Ex_i = \frac{\delta_i^{\max} + \beta_i^{\min}}{2}, \tag{2}$$

$$En_i = \frac{\delta_i^{\max} - \beta_i^{\min}}{6}, \tag{3}$$

and

$$He = k,$$

$k$ being a constant, and generally $He = 0.1En_i$.

The cloud model describes the randomness and fuzziness of evaluation language sets vividly, but in order to facilitate the actual operation, further fitted curve is needed. Based on the atomization characteristics of normal cloud model, when $He_i < \frac{1}{3}En_i$, 99.7 % of the cloud drop will be within the area of the maximum boundary curve $y_1 = \exp\left[-(x - Ex)^2 / 2(En + 3He)^2\right]$ and minimum boundary curve $y_1 = \exp\left[-(x - Ex)^2 / 2(En - 3He)^2\right]$. Therefore, this paper, mid-expect curve is used in calculation, instead of the remark set cloud model, as shown in Fig. 2.

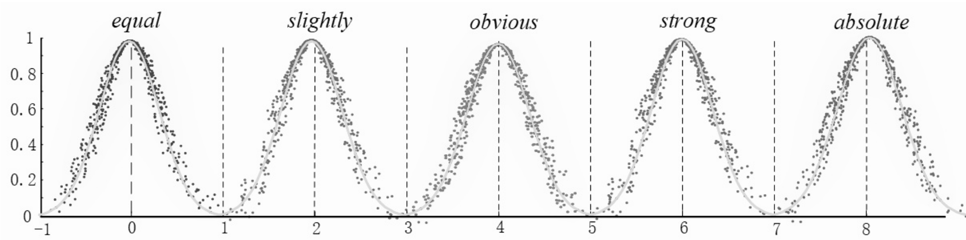


Fig. 2. Fitted curve sketch map of pairwise comparison

People tend to evaluate the importance degree by using expectations and further determine the deviation degree and growing trend from the important degree to the rating grade expectations by using membership degree. Composite evaluation scale expressions composed of expectations and membership degree is shown as follows:

$$y_i = \begin{cases} \delta_i^{\max} - \mu_i & x_i > Ex_i \geq 0 \\ \frac{\delta_i^{\max} + \beta_i^{\min}}{2} & x_i = Ex_i \\ \beta_i^{\min} + \mu_i & 0 < x_i < Ex_i \end{cases} . \tag{4}$$

As shown in Fig. 2, assume that there are five importance evaluation levels in one qualitative concept, namely equally, slightly, obviously strong and absolute. In order to better describe the things, this paper uses the median curve of the two boundary curves $y_i = \exp\left[(x_i - Ex_i)^2 / \left(2(En)^2\right)\right]$ instead of the set of cloud models to participate in the calculation. The value range of membership $y_i$ is [0, 1] and on the basis of the evaluation level, people's subjective evaluation of things can be further refined.

## 3. Determination of importance ratio parameter $a$

While constructing judgment matrix by index cloud composite scale, the $a$ value represents the objective importance ratio ($a > 1$) between the adjacent two levels, which can satisfy the requirement of consistency check, and can conform to the actual situation [5], [6], [9], [10]. On the basis of existing achievements, from the changing trend of computational consistency index C.L., calculation consistency proportion C.R. and index weight difference, this paper proposes a basis to determine the ratio of index scale $a$.

Suppose now equipment purchase is needed, $B_1$ is for quality, $B_2$ price and $B_3$ after service, and three factories, $C_1$, $C_2$ and $C_3$ are all available. First of all, set up a hierarchical structure, as shown in Fig. 3, then experts help to establish judgment matrix at different levels, as shown in Table [1]. When $a \in [1.05, 1.7]$, examine whether the judgment matrix can meet the consistency of important degree and determine whether the results meet the practical situation or not.

Table 1. Judgment matrix at different levels

| **A** | **B1** | **B2** | **B3** | **B1** | **C1** | **C2** | **C3** |
|---|---|---|---|---|---|---|---|
| **B1** | $a^{0}$ | $a^{2.3}$ | $a^{6.7}$ | **C 1** | $a^{0}$ | $a^{2.7}$ | $a^{2.2}$ |
| **B2** | $a^{-2.3}$ | $a^{0}$ | $a^{4.6}$ | **C 2** | $a^{-2.7}$ | $a^{0}$ | $a^{0}$ |
| **B3** | $a^{-6.7}$ | $a^{4.6}$ | $a^{0}$ | **C 3** | $a^{-2.2}$ | $a^{0}$ | $a^{0}$ |
| **B2** | **C1** | **C2** | **C3** | **B3** | **C1** | **C2** | **C3** |
| **C1** | $a^{0}$ | $a^{-4.6}$ | $a^{2.7}$ | **C1** | $a^{0}$ | $a^{2.4}$ | $a^{6.6}$ |
| **C2** | $a^{4.6}$ | $a^{0}$ | $a^{4.2}$ | **C2** | $a^{-2.4}$ | $a^{0}$ | $a^{4.4}$ |
| **C3** | $a^{-2.7}$ | $a^{-4.2}$ | $a^{0}$ | **C3** | $a^{-6.6}$ | $a^{-4.4}$ | $a^{0}$ |

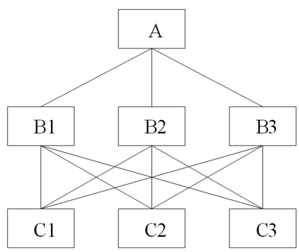


Fig. 3. Hierarchical structure

# 4. The consistency check of sensory judgment matrix

## 4.1. Consistency improvement of sensory judgment matrix

**Theorem 1.** When sensory judgment matrix is completely consistent, then

$$\begin{cases} c_{ij} = c_{i(i+1)} + c_{(i+1)(i+2)} + \cdots + c_{(j-1)j} \\ c_{ji} = -c_{ij} \end{cases} \quad (i > j)\,. \tag{5}$$

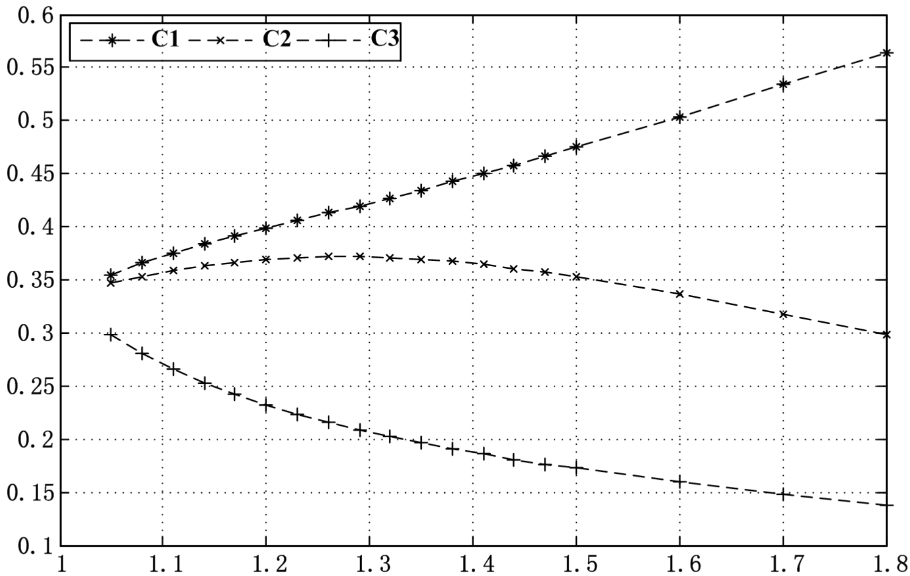


Fig. 4. Weight variation curve of $C_1$, $C_2$ and $C_3$ with different $a$

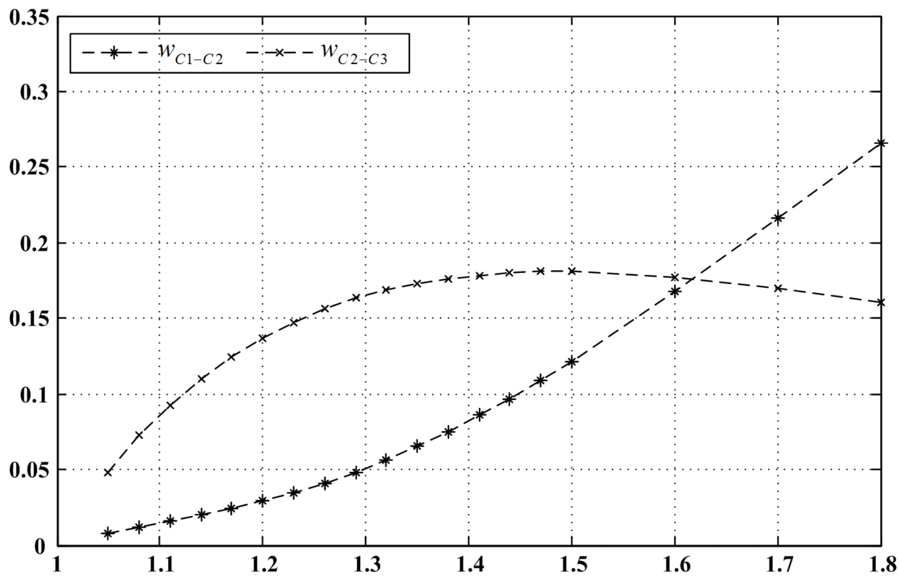


Fig. 5. Weight difference curve of $C_1 - C_2$ and $C_2 - C_3$ with different $a$

In other words, when sensory judgment matrix is completely consistent, each element can be obtained from $n-1$ data, which are derived from comparison of two factors in between.

**Certify:** if reciprocal judgment matrix is fully consistent, then

$$\alpha_{ij} = a^{c_{ij}} = \frac{\omega_i}{\omega_j} = \frac{\omega_i}{\omega_{i+1}} \frac{\omega_{i+1}}{\omega_{i+2}} \cdots \frac{\omega_{j-1}}{\omega_j} = a^{c_{i(i+1)}} a^{c_{(i+1)(i+2)}} \cdots a^{c_{(j-1)j}} =$$

$$= a^{c_{i(i+1)}+c_{(i+1)(i+2)}+\cdots c_{(j-1)j}} . \tag{6}$$

According to theorem 1, completely consistent sensory judgment matrix can be achieved from the diagonal elements of initial sensory judgment matrix. However, it is possible that tiny deviations of the diagonal elements will result in subjective judgment deviation, and thereby, review and adjust will be necessary to avoid further deviation. In this way, it is the same with the following digital programming problem:

$$\min \left[ \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left( c_{ij} - \sum_{m=i}^{j-1} x_m \right)^2 \right] , \tag{7}$$

s.t.

$$\begin{cases} \left| c_{ij} - \sum\limits_{m=i}^{j-1} x_m \right| \le 0.5, \; j > i \\ x_m \in R, \; m = 1, 2, \cdots, n-1 \end{cases} .$$

Here, the objective function ensures that the gap between full consistency matrix constructed by $X$ and reformed initial sensory judgment matrix as small as possible, which can be no more than 0.5, and $X_0 = \left(c_{i(i+1)}\right)_{1\times(n-1)}$ $(i = 1, 2, \cdots, n-1)$.

To sum up, consistency improvement of sensory judgment matrix and algorithm concerning the weight calculation based on the cloud index scale method is presented as follows:

1. Experts give the initial sensory judgment matrix $\boldsymbol{C}$.
2. Assign $c_{i(i+1)}$ $(i = 1, 2, \cdots, n-1)$ to $X_0 = \left(x_m^0\right)_{1\times(n-1)}$.
3. To avoid the psychological distinguish limit 9, transformation of initial sensory judgment matrix is done, if $\sum\limits_{m=i}^{j-1} x_m^0 > 8$, $c_{ij} = \sum\limits_{m=i}^{j-1} x_m^0$ then $\boldsymbol{C}'$ can be got.
4. Put $\boldsymbol{C}'$ into mathematical programming problem like (8).
5. Assume the initial answer to the question above to be $X_0$, optimum solution $X^\star$.
6. Get full consistency sensory judgment matrix $\boldsymbol{C}^\star$ based on theorem1.
7. Define importance rate $a$, convert full consistency sensory judgment matrix $\boldsymbol{C}^\star$ into objective difference judgment matrix based on (5).

8. Select any column of objective difference judgment matrix, the attribute weights vector will be obtained when normalized.

### 4.2. Examples of algorithmic demonstrating

Assume that there are 4 index namely $c_1$, $c_2$, $c_3$, $c_4$ in an evaluation system, the weight of which is required. Here, AHP weighting algorithm based on cloud index composite scale is adopted.

1. Experts give the initial sensory judgment matrix $\boldsymbol{C}$.

$$\boldsymbol{C} = \begin{bmatrix} 0 & 2.4 & 4.2 & 1.7 \\ -2.4 & 0 & 2.6 & -1.8 \\ -4.2 & -2.6 & 0 & -2.2 \\ -1.7 & 1.8 & 2.2 & 0 \end{bmatrix} \boldsymbol{C}^{\star} = \begin{bmatrix} 0 & 2.4 & 5.05 & 2.9 \\ -2.4 & 0 & 2.65 & 0.5 \\ -5.05 & -2.65 & 0 & -2.15 \\ -2.9 & -0.5 & 2.15 & 0 \end{bmatrix}. \tag{8}$$

2. Assume $x_1^0 = \boldsymbol{C}(1,2) = 2.4$, $x_2^0 = \boldsymbol{C}(2,3)$, $x_3^0 = \boldsymbol{C}(3,4) = -2.2$.

3. If $\sum\limits_{m=i}^{j-1} x_m^0 > 8$, $c_{ij} = \sum\limits_{m=i}^{j-1} x_m^0$, then $\boldsymbol{C}' = \boldsymbol{C}$, and put $\boldsymbol{C}'$ into the following (8) and use Quadprog in MATLAB to deal with the question, then we get $X^{\star} = (2.4, 2.65, -2.15)$, so get full consistency sensory judgment matrix $\boldsymbol{C}^{\star}$ based on theorem1.

4. Select importance rate $a = 1.3$, build full consistency objective judgment matrix $A$ based on (1), then normalize the last column of $A$, then target weight $w$ is obtained

$$A = \begin{bmatrix} 1.0000 & 1.8770 & 3.7620 & 2.1401 \\ 0.5328 & 1.0000 & 2.0042 & 1.1402 \\ 0.2658 & 0.4989 & 1.0000 & 0.5689 \\ 0.4673 & 0.8771 & 1.7578 & 1.0000 \end{bmatrix},$$

$$w = \begin{bmatrix} 0.4413 & 0.2351 & 0.1173 & 0.2062 \end{bmatrix}. \tag{9}$$

5. Efficacy check: Concordance index: $CI = -2.9606e - 016$; consistency ratio $CR = 0.9 < 01$, thus the conclusion is effective.

## 5. Summary

This paper proposes a cloud index composite scale method and its optimization strategy concerning language assessment information, which makes possible accurate expression of language information based on traditional ways, and extends the application of scale weightiness. At the same time, in order to facilitate the actual application of cloud index composite scale method, another two issues are also discussed in this paper: (1) the determination of the importance ratio parameter $a$; (2)

completely consistent sensory judgment matrix and the algorithm concerning index weight accordingly. The following conclusions can be drowned based on previous discussions:

1. Results of each judgment matrix constructed by the cloud index composite scale vary from the value $a$. In order to satisfy the demands of higher accuracy, when degree of differentiation is the maximum, the best value $a$ should be near the extreme value point. If it is a monotone curve with no extreme value point, then value $a$ has no limitation, while considering the practical situation, value $a$ should be between 1.1 and 1.5.

2. The objective differences judgment matrix obtained through the sensory judgment matrix is reciprocal judgment matrix, which meet the demand of the consistency check and the consistency and transitivity of important degree.

3. When dimension of the sensory judgment matrix is bigger, namely when there are relatively more properties in a judgment, transform initial sensory judgment matrix into a quadratic programming problem will be advisable, since people think so differently that objective judgment matrix is hard to meet the completely consistency.

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Received April 30, 2017